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Stability of a class of networked control systems with Markovian characterization

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ABSTRACT

This paper studies the stability problem for a class of networked control systems (NCSs) with the plant being a Markovian jump system. The random delays from the sensor to the controller and from the controller to the actuator are modeled as two Markov chains. The necessary and sufficient conditions for the stochastic stability are established. The state-feedback controller gain that depends on not only the delay modes but also the system mode is obtained through the iterative linear matrix inequality approach. An illustrative example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

With the very rapid advances in communication networks, networked control systems (NCSs) whose control loops are connected via communication networks have attracted increasing attention in the recent years [1–7]. It is known that time delays always occur when sensors, actuators, and controllers exchange data through networks and the occurrence of time delays may cause systems instability, therefore, many researchers have studied the stability problem of NCSs with time delays [8,9]. In many cases, the time delays are random and can be modeled as Markov chains [10–14]. In [10], the state-feedback controller gain is mode-independent. In [11,12], to reduce the conservativeness of the stabilization condition of an NCS, a mode-dependent state-feedback controller gain is designed. Unfortunately, only the sensor-to-controller (S–C) delay is considered. In [13,14], the controller depends on the S–C delay mode as well as the controller-to-actuator (C–A) delay mode, however, it is worthwhile noting that the plant is a deterministic system.

On the other hand, in the past several years, increasing efforts have been devoted to stochastic systems, especially Markovian jump systems which have transition between models determined by a Markov chain. Markovian jump systems can be applied in many areas, e.g. aircraft control, manufacturing systems and sliding mode control. The researches of the stability of Markovian jump systems can be seen widely in the literature [15–18]. It should be pointed out that the results of the aforementioned references cannot be applied to NCSs.

So far, the stability synthesis for the NCSs with the plant being a Markovian jump system has not been fully investigated, especially for the NCSs whose state-feedback controller gain depends on the pant mode and the S–C and C–A delays. To the best of the authors' knowledge, few results have been available in the literature.

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In this paper, the stability problem for a class of NCSs with a discrete-time Markovian jump plant is investigated. By considering the random delays in the S–C and C–A sides, the state-feedback closed-loop NCS is modeled as a Markovian jump system. The necessary and sufficient conditions for the stochastic stability are derived. An iterative linear matrix inequality approach is employed to design the three-mode-dependent state-feedback controller gain. A numerical example verifies its effectiveness.

2. Model for networked control system

The structure of the NCS is shown in Fig. 1. The discrete-time Markovian jump system is described by

$$x(k+1) = A(\theta_k)x(k) + B(\theta_k)u(k), \tag{1}$$

where $k \in Z_+$, $x(k) \in R^n$ is the state, $u(k) \in R^m$ is the control input, θ_k denotes the system mode. The matrices $A_i = A(\theta_k = i)$, $B_i = B(\theta_k = i)$ are constant matrices of appropriate dimensions. As shown in Fig. 1, the random delays exist in the S–C and C–A sides. Here τ_k represents the S–C delay, and d_k stands for the C–A delay, θ_k, τ_k and d_k are modeled as three homogeneous Markov chains that take values in $\Im = \{1, 2, \ldots, \eta\}$, $\Re = \{0, 1, \ldots, \tau\}$ and $\aleph = \{0, 1, \ldots, d\}$, $\eta \in Z_+$, $\tau, d \in N$. The transition probability matrices of θ_k, τ_k and d_k are $\Lambda = [\omega_{ij}]$, $\Gamma = [\lambda_{lh}]$ and $\Pi = [\pi_{rs}]$, respectively. That means θ_k, τ_k and d_k jump from mode i to j, from mode l to h and from mode r to s, respectively, with probabilities ω_{ij}, λ_{lh} and π_{rs}:

$$\omega_{ij} = P(\theta_{k+1} = j | \theta_k = i), \quad \lambda_{lh} = P(\tau_{k+1} = h | \tau_k = l), \quad \pi_{rs} = P(d_{k+1} = s | d_k = r), \tag{2}$$

where ω_{ij}, λ_{lh}, $\pi_{rs} \geqslant 0$ and

$$\sum_{j=1}^{\eta} \omega_{ij} = 1, \quad \sum_{h=0}^{\tau} \lambda_{lh} = 1, \quad \sum_{s=0}^{d} \pi_{rs} = 1. \tag{3}$$

In the NCSs, the delay information is important for the controller design. By using the embedded processor and time-stamping technique [14], the information of $d_{k-\tau_k-1}$ at time instant k is known at the controller node if the time delay τ_k exits. By considering the effect of random delays, the mode-dependent state-feedback controller is designed as

$$u(k) = K\left(\theta_{k-\tau_k-d_{k-1-\tau_k}}\right)x(k-\tau_k-d_k). \tag{4}$$

Applying (4)–(1) gives the closed-loop system

$$x(k+1) = A(\theta_k)x(k) + B(\theta_k)K(\theta_{k-\tau_k-d_{k-1-\tau_k}})x(k-\tau_k-d_k). \tag{5}$$

Define

$$\begin{aligned}
&\xi(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & x^{\mathrm{T}}(k-1) & \cdots & x^{\mathrm{T}}(k-\tau-d) \end{bmatrix}^{\mathrm{T}}, \\
&\tilde{\theta}_k = [\theta_k \quad \theta_{k-1} \quad \cdots \quad \theta_{k-\tau-d}]^{\mathrm{T}}, \\
&\widehat{K}(\tilde{\theta}_k) = \begin{bmatrix} K^{\mathrm{T}}(\theta_k) & K^{\mathrm{T}}(\theta_{k-1}) & \cdots & K^{\mathrm{T}}(\theta_{k-\tau-d}) \end{bmatrix}^{\mathrm{T}}, \\
&R_1(\tau_k + d_{k-1-\tau_k}) = [0_{m\times m} \quad \cdots \quad 0_{m\times m} \quad I_{m\times m} \quad 0_{m\times m} \quad \cdots \quad 0_{m\times m}] \in R^{m\times(d+\tau+1)m}, \\
&R_2(\tau_k + d_k) = [0_{n\times n} \quad \cdots \quad 0_{n\times n} \quad I_{n\times n} \quad 0_{n\times n} \quad \cdots \quad 0_{n\times n}] \in R^{n\times(d+\tau+1)n},
\end{aligned} \tag{6}$$

where $I_{m\times m}$ is the $(\tau_k + d_{k-1-\tau_k} + 1)$th of $R_1(\tau_k + d_{k-1-\tau_k})$ and $I_{n\times n}$ is the $(\tau_k + d_k + 1)$th of $R_2(\tau_k + d_k)$.

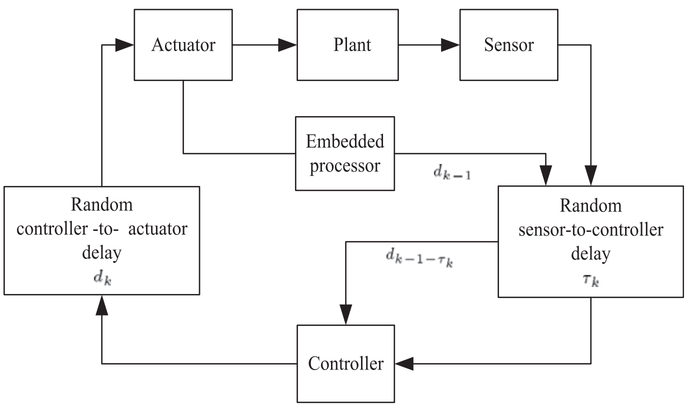

Fig. 1. The structure of NCS.

From (5) and (6), we obtain the closed-loop system

$$\xi(k+1) = [\widehat{A}(\tilde{\theta}_k) + \widehat{B}(\tilde{\theta}_k)R_1(\tau_k + d_{k-1-\tau_k})\widehat{K}(\tilde{\theta}_k)R_2(\tau_k + d_k)]\xi(k), \tag{7}$$

where

$$\widehat{A}(\tilde{\theta}_k) = \begin{bmatrix} A(\theta_k) & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad \widehat{B}(\tilde{\theta}_k) = \begin{bmatrix} B(\theta_k) \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Definition 1. The closed-loop system in (7) is stochastically stable if for every initial condition $\xi_0 = \xi(0), \tilde{\theta}_0 \in \Im \times \Im \times \cdots \times \Im$, $\tau_0 \in \Re$, and $d_{-\tau_0-1} \in \aleph$, there exists a matrix $W > 0$ such that

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|\xi(k)\|^2 | \xi_0, \tilde{\theta}_0, \tau_0, d_{-\tau_0-1}\right\} < \xi_0^{\mathrm{T}} W \xi_0. \tag{8}$$

3. Main results

The following lemma is introduced which is useful for the development of this work.

Lemma 1 [19]. *Given $d \in N$, $\tau \in N$ and two sets $\Im^{\tau+d+1} = \Im \times \Im \times \cdots \times \Im$ and $\Im_{\tau+d+1} = \{1, 2, \cdots, \eta^{\tau+d+1}\}$, introduce the mapping $\psi : \Im^{\tau+d+1} \to \Im_{\tau+d+1}$ with*

$$\psi(\chi) = i + (i_{-1} - 1)\eta + (i_{-2} - 1)\eta^2 \cdots + (i_{-\tau-d} - 1)\eta^{\tau+d}, \tag{9}$$

where $i, i_{-1}, \cdots, i_{-\tau-d} \in \Im$ and $\chi = [\,i \quad i_{-1} \quad i_{-2} \quad \cdots \quad i_{-\tau-d}\,]^T \in \Im^{\tau+d+1}$. Then, the mapping $\psi(\cdot)$ is a bijection from $\Im^{\tau+d+1}$ to $\Im_{\tau+d+1}$.

Since $\psi(\cdot)$ is a bijection, for any arbitrary two modes $\nu,\ \mu \in \Im_{\tau+d+1}$, we can uniquely obtain two vectors $\tilde{\nu},\ \tilde{\mu} \in \Im^{\tau+d+1}$, such that

$$\tilde{\nu} = \psi^{-1}(\nu) = [\,i \quad i_{-1} \quad i_{-2} \quad \cdots \quad i_{-\tau-d}\,]^{\mathrm{T}},$$

$$\tilde{\mu} = \psi^{-1}(\mu) = [\,j \quad j_{-1} \quad j_{-2} \quad \cdots \quad j_{-\tau-d}\,]^{\mathrm{T}}. \tag{10}$$

By letting $\tilde{\theta}_{k+1} = \tilde{\mu}, \tilde{\theta}_k = \tilde{\nu}$, it can be seen that the transition probability $P(\tilde{\theta}_{k+1} = \tilde{\mu} | \tilde{\theta}_k = \tilde{\nu})$ is nonzero if:

$$\theta_k = i = j_{-1}, \quad \theta_{k-1} = i_{-1} = j_{-2}, \ldots, \quad \theta_{k-\tau-d+1} = i_{-\tau-d+1} = j_{-\tau-d}. \tag{11}$$

Then, we have

$$\begin{aligned} P\Big(\psi(\tilde{\theta}_{k+1}) = \mu | \psi\Big(\tilde{\theta}_k\Big) = \nu\Big) &= P\Big(\tilde{\theta}_{k+1} = \tilde{\mu} | \tilde{\theta}_k = \tilde{\nu}\Big) \\ &= P(\theta_{k+1} = j, \theta_k = j_{-1}, \ldots, \theta_{k-\tau-d+1} = j_{-\tau-d} | \theta_k = i, \theta_{k-1} = i_{-1}, \ldots, \theta_{k-\tau-d} = i_{-\tau-d}) \\ &= \omega_{ij}\delta(i, j_{-1})\delta(i_{-1}, j_{-2}) \cdots \delta(i_{-\tau-d+1}, j_{-\tau-d}), \end{aligned} \tag{12}$$

where

$$\delta(p, q) = \begin{cases} 0 & \text{if } p \neq q, \\ 1 & \text{if } p = q. \end{cases}$$

Based upon the Lemma 1 and the above analyse, we give the necessary and sufficient conditions for the stochastic stability of system in (7).

Theorem 1. *Closed-loop system (7) is stochastically stable if and only if there exist matrices $Q(\nu, l, r) > 0$ and $K_{i_{-l-r}}$ of appropriate dimensions such that the following inequality holds*

$$\begin{aligned} L(\nu, l, r) = &\sum_{j=1}^{\eta} \sum_{h=0}^{\tau} \sum_{s_2=0}^{d} \sum_{s_1=0}^{d} \omega_{ij}\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_2 s_1}^{h} \times \Big[\widehat{A}_\nu + \widehat{B}_\nu K_{i_{-l-r}} R_2(l+s_1)\Big]^{\mathrm{T}} Q(\beta + j, h, s_2) \times [\widehat{A}_\nu + \widehat{B}_\nu K_{i_{-l-r}} R_2(l+s_1)] \\ &- Q(\nu, l, r) < 0 \end{aligned} \tag{13}$$

for all $l \in \Re, r \in \aleph, v \in \Im_{\tau+d+1}$, *where*

$$K_t = K(t), \quad \psi^{-1}(v) = [\, i \quad i_{-1} \quad i_{-2} \quad \cdots \quad i_{-\tau-d}\,]^{\mathrm{T}},$$

$$\widehat{A}_v = \begin{bmatrix} A_i & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad \widehat{B}_v = \begin{bmatrix} B_i \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

$$\beta = (i-1)\eta + (i_{-1}-1)\eta^2 + (i_{-2}-1)\eta^3 + \cdots + (i_{-\tau-d+1}-1)\eta^{\tau+d}. \tag{14}$$

Proof. Sufficiency: For system (7), construct the Lyapunov function

$$V(\xi(k),k) = \xi^{\mathrm{T}}(k)Q(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1})\xi(k).$$

Then, we have

$$\begin{aligned}\mathcal{E}\{\triangle V(\xi(k),k)\} &= \mathcal{E}\left\{V(\xi(k+1),k+1)|\xi(k),\psi\left(\tilde{\theta}_k\right),\tau_k,d_{k-\tau_k-1}\right\} - V(\xi(k),k) \\ &= \mathcal{E}\left\{\xi^{\mathrm{T}}(k+1)Q\left(\psi(\tilde{\theta}_{k+1}),\tau_{k+1},d_{k-\tau_{k+1}}\right)\xi(k+1)|\xi(k),\psi\left(\tilde{\theta}_k\right),\tau_k,d_{k-\tau_k-1}\right\} \\ &\quad - \left\{\xi^{\mathrm{T}}(k)Q\left(\psi\left(\tilde{\theta}_k\right),\tau_k,d_{k-\tau_k-1}\right)\xi(k)\right\}.\end{aligned}$$

Let

$$\begin{aligned}&\tau_k = l, \quad \tau_{k+1} = h, \quad d_{k-\tau_k-1} = r, \quad d_{k-\tau_{k+1}} = s_2, \quad d_k = s_1, \\ &\psi(\tilde{\theta}_k) = v = i + (i_{-1}-1)\eta + (i_{-2}-1)\eta^2 + \cdots + (i_{-\tau-d}-1)\eta^{\tau+d}, \\ &\psi(\tilde{\theta}_{k+1}) = \mu = j + (j_{-1}-1)\eta + (j_{-2}-1)\eta^2 + \cdots + (j_{-\tau-d}-1)\eta^{\tau+d}.\end{aligned} \tag{15}$$

Then, the probability transition matrices are

$$\tau_k \to \tau_{k+1} : \Gamma, \quad d_{k-\tau_k-1} \to d_{k-\tau_{k+1}} : \Pi^{1+l-h}, \quad d_{k-\tau_{k+1}} \to d_k : \Pi^h \tag{16}$$

and

$$\begin{aligned}&\widehat{K}(\tilde{\theta}_k) = [\,K_i^{\mathrm{T}} \quad K_{i_{-1}}^{\mathrm{T}} \quad \cdots \quad K_{i_{-l-r}}^{\mathrm{T}} \quad \cdots \quad K_{i_{-\tau-d}}^{\mathrm{T}}\,]^{\mathrm{T}}, \\ &R_1(\tau_k + d_{k-1-\tau_k})\widehat{K}(\tilde{\theta}_k) = K_{i_{-l-r}}, \quad R_2(\tau_k + d_k) = R_2(l+s_1), \\ &\widehat{A}(\tilde{\theta}_k) = \widehat{A}_v, \quad \widehat{B}(\tilde{\theta}_k) = \widehat{B}_v.\end{aligned} \tag{17}$$

From (12), (14)–(17), we have

$$\begin{aligned}\mathcal{E}\{\triangle V(\xi(k),k)\} =&\xi^{\mathrm{T}}(k)\left\{\sum_{j=1}^{\eta}\sum_{h=0}^{\tau}\sum_{s_2=0}^{d}\sum_{s_1=0}^{d}\omega_{ij}\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_2s_1}^{h} \times [\widehat{A}_v + \widehat{B}_vK_{i_{-l-r}}R_2(l+s_1)]^{\mathrm{T}}Q(\beta+j,h,s_2)\right. \\ &\left.\times[\widehat{A}_v + \widehat{B}_vK_{i_{-l-r}}R_2(l+s_1)] - Q(v,l,r)\right\}\xi(k) = \xi^{\mathrm{T}}(k)L(v,l,r)\xi(k).\end{aligned}$$

Thus if $L(v,l,r) < 0$, then

$$\mathcal{E}\{\triangle V(\xi(k),k)\} \leqslant -\lambda_{min}(-L(v,l,r))\xi^{\mathrm{T}}(k)\xi(k) \leqslant -\alpha\|\xi(k)\|^2, \tag{18}$$

where $\alpha = inf\{\lambda_{min}(-L(v,l,r))\} > 0$.

It follows from (18) that for any $n \geqslant 1$

$$\mathcal{E}\{V(\xi(n+1),n+1)\} - \mathcal{E}\{V(\xi(0),0)\} \leqslant -\alpha\mathcal{E}\left(\sum_{m=0}^{n}\|\xi(m)\|^2\right). \tag{19}$$

Furthermore, we have

$$\mathcal{E}\left(\sum_{m=0}^{n}\|\xi(m)\|^2\right) \leqslant \frac{1}{\alpha}(\mathcal{E}\{V(\xi(0),0)\} - \mathcal{E}\{V(\xi(n+1),n+1)\}) \leqslant \frac{1}{\alpha}\mathcal{E}\{V(\xi(0),0)\} \leqslant \frac{1}{\alpha}\xi^{\mathrm{T}}(0)Q\left(\psi(\tilde{\theta}_0),\tau_0,d_{-\tau_0-1}\right)\xi(0). \tag{20}$$

By using Definition 1, the closed-loop system in (7) is stochastically stable.

Necessity: Assume that $G(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1}) > 0$ and define

$$\xi^{\mathrm{T}}(t)\widetilde{Q}\Big(T-t, \psi\Big(\tilde{\theta}_t\Big), \tau_t, d_{t-\tau_t-1}\Big)\xi(t) = \mathcal{E}\left\{\sum_{k=t}^{T}\xi^{\mathrm{T}}(k)G\Big(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1}\Big)\xi(k)|\xi_t, \psi\Big(\tilde{\theta}_t\Big), \tau_t, d_{t-\tau_t-1}\right\}. \tag{21}$$

Since $G\Big(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1}\Big) > 0$, as T increases, $\xi^{\mathrm{T}}(t)\widetilde{Q}(T-t, \psi(\tilde{\theta}_t), \tau_t, d_{t-\tau_t-1})\xi(t)$ is increasing. From (8), we get that $\xi^{\mathrm{T}}(t)\widetilde{Q}(T-t, \psi(\tilde{\theta}_t), \tau_t, d_{t-\tau_t-1})\xi(t)$ is upper bounded. Furthermore, its limit exits and can be expressed as

$$\begin{aligned}\xi^{\mathrm{T}}(t)Q(v,l,r)\xi(t) &= \lim_{T\to\infty}\xi^{\mathrm{T}}(t)\widetilde{Q}(T-t, \psi(\tilde{\theta}_t) = v, \tau_t = l, d_{t-\tau_t-1} = r)\xi(t)\\ &= \lim_{T\to\infty}\mathcal{E}\left\{\sum_{k=t}^{T}\xi^{\mathrm{T}}(k)G(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1})\xi(k)|\xi_t, \psi(\tilde{\theta}_t) = v, \tau_t = l, d_{t-\tau_t-1} = r\right\}.\end{aligned} \tag{22}$$

Thus, we have

$$Q(v,l,r) = \lim_{T\to\infty}\widetilde{Q}\Big(T-t, \psi(\tilde{\theta}_t) = v, \tau_t = l, d_{t-\tau_t-1} = r\Big). \tag{23}$$

From (22) and noticing that $G(\psi(\tilde{\theta}_k), \tau_k, d_{k-\tau_k-1}) > 0$, we obtain $Q(v,l,r) > 0$.

From (21), we have

$$\begin{aligned}&\mathcal{E}\Big\{\xi^{\mathrm{T}}(t)\widetilde{Q}(T-t, \psi(\tilde{\theta}_t), \tau_t, d_{t-\tau_t-1})\xi(t) - \xi^{\mathrm{T}}(t+1)\widetilde{Q}(T-t-1, \psi\Big(\tilde{\theta}_{t+1}\Big), \tau_{t+1}, d_{t-\tau_{t+1}})\xi(t+1)|\xi_t,\ \psi(\tilde{\theta}_t) = v, \tau_t = l, d_{t-\tau_t-1} = r\Big\}\\ &\quad = \xi^{\mathrm{T}}(t)G(v,l,r)\xi(t).\end{aligned} \tag{24}$$

From (7) and (16), we obtain

$$\begin{aligned}&\mathcal{E}\Big\{\xi^{\mathrm{T}}(t)\widetilde{Q}(T-t, \psi(\tilde{\theta}_t), \tau_t, d_{t-\tau_t-1})\xi(t) - \xi^{\mathrm{T}}(t+1)\widetilde{Q}(T-t-1, \psi(\tilde{\theta}_{t+1}), \tau_{t+1}, d_{t-\tau_{t+1}})\xi(t+1)|\xi_t, \psi(\tilde{\theta}_t) = v, \tau_t = l, d_{t-\tau_t-1} = r\Big\}\\ &\quad = \xi^{\mathrm{T}}(t)\Bigg\{\widetilde{Q}(T-t, v, l, r) - \sum_{j=1}^{\eta}\sum_{h=0}^{\tau}\sum_{s_2=0}^{d}\sum_{s_1=0}^{d}\omega_{ij}\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_2s_1}^{h}\times[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+s_1)]^{\mathrm{T}}\times\widetilde{Q}(T-t-1, \beta+j, h, s_2)\\ &\qquad\times[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+s_1)]\Bigg\}\xi(t).\end{aligned} \tag{25}$$

It is easy to obtain from (24) and (25) that

$$\begin{aligned}&\xi^{\mathrm{T}}(t)\Bigg\{\widetilde{Q}(T-t, v, l, r) - \sum_{j=1}^{\eta}\sum_{h=0}^{\tau}\sum_{s_2=0}^{d}\sum_{s_1=0}^{d}\omega_{ij}\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_2s_1}^{h}\times[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+s_1)]^{\mathrm{T}}\times\widetilde{Q}(T-t-1, \beta+j, h, s_2)\\ &\quad\times[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+s_1)]\Bigg\}\xi(t) = \xi^{\mathrm{T}}(t)G(v,l,r)\xi(t).\end{aligned} \tag{26}$$

Let $T\to\infty$ and from (23), we obtain that (13) holds. This completes the proof. □

Theorem 1 gives the necessary and sufficient conditions on the existence of the mode-dependent state-feedback stabilizing gain. Based on the results in Theorem 1, the controller design techniques are given in Theorem 2.

Theorem 2. *There exists a controller in (4) such that the system in (7) is stochastically stable if and only if there exits matrices $Q(v,l,r) > 0$ and $K_{i\_l\_r}$ of appropriate dimensions satisfying*

$$\begin{bmatrix}-Q(v,l,r) > 0 & W^{\mathrm{T}}(v,l,r)\\ W(v,l,r) & -X(v,l,r)\end{bmatrix} < 0$$

for all $l\in\Re, r\in\aleph, v\in\Im_{\tau+d+1}$, and

$$W(v,l,r) = [\,W_0^{\mathrm{T}}\quad W_1^{\mathrm{T}}\quad\cdots\quad W_\tau^{\mathrm{T}}\,]^{\mathrm{T}}\quad W_h = \Big[\,W_{h,0}^{\mathrm{T}}\quad W_{h,1}^{\mathrm{T}}\quad\cdots\quad W_{h,d}^{\mathrm{T}}\,\Big]^{\mathrm{T}}, \tag{27}$$

$$W_{h,s_2} = \begin{bmatrix}\Big(\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_20}^{h}\Big)^{\frac{1}{2}}\Big[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+0)\Big]\\ \Big(\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_21}^{h}\Big)^{\frac{1}{2}}\Big[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+1)\Big]\\ \vdots\\ \Big(\lambda_{lh}\Pi_{rs_2}^{1+l-h}\Pi_{s_2d}^{h}\Big)^{\frac{1}{2}}\Big[\widehat{A}_v + \widehat{B}_vK_{i\_l\_r}R_2(l+d)\Big]\end{bmatrix}, \tag{28}$$

$$X(v,l,r) = diag\{X_0\quad X_1\quad\cdots\quad X_\tau\}\quad X_h = diag\{X_{h,0}\quad X_{h,1}\quad\cdots\quad X_{h,d}\}, \tag{29}$$

$$X_{h,s_2} = diag\{\underbrace{\overline{X}_{hs_2} \quad \overline{X}_{hs_2} \quad \cdots \quad \overline{X}_{hs_2}}_{d+1}\} \quad \overline{X}_{hs_2} = \left[\sum_{j=1}^{\eta} \omega_{ij} Q(\beta + j, h, s_2)\right]^{-1}. \tag{30}$$

Proof. By applying the Schur complement and letting $\overline{X}_{hs_2} = \left[\sum_{j=1}^{\eta} \omega_{ij} Q(\beta + j, h, s_2)\right]^{-1}$, the proof can be easily obtained.

The conditions in Theorem 2 are a set of linear matrix inequalities with some inversion constraints. $K_{i_{-l-r}}$ can be solved by an iterative linear matrix inequality approach which is called as the cone complementarity linearization algorithm (see [20,21] for details). □

4. Numerical example

Consider the following system in [12]

$$x(k+1) = A(\theta_k)x(k) + B(\theta_k)u(k) \tag{31}$$

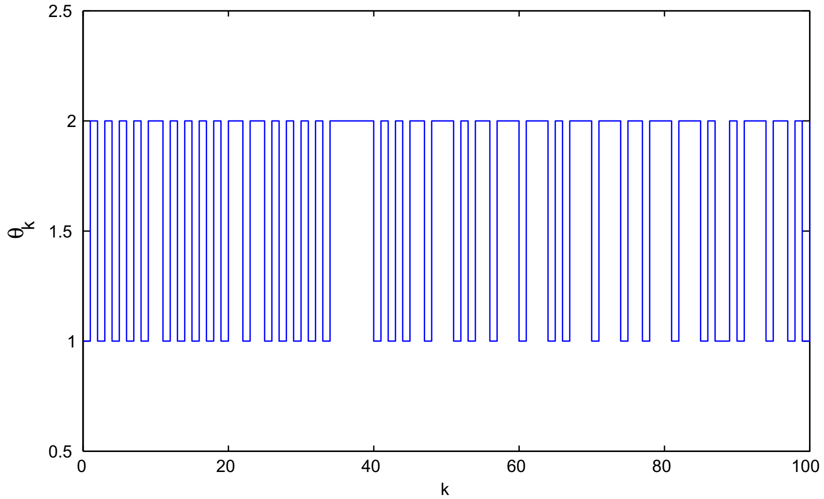

Fig. 2. Random mode θ_k.

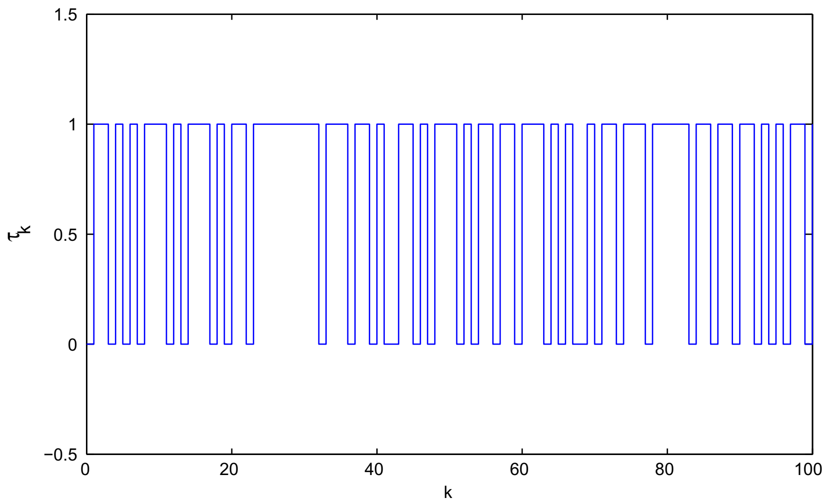

Fig. 3. Random mode τ_k.

where

$$\theta_k \in \Im = \{1,2\}, \quad \tau_k \in \Re = \{0,1\}, \quad d_k \in \aleph = \{0,1\}.$$
$$A_1 = \begin{bmatrix} 1.7 & 1 \\ 0 & 0.17 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0.6 & 0 \\ 0.04 & 2 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\Lambda = \begin{bmatrix} 0.1 & 0.9 \\ 0.5 & 0.5 \end{bmatrix} \quad \Gamma = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0.1 & 0.9 \\ 0.8 & 0.2 \end{bmatrix}. \tag{32}$$

Thus we have $\eta = 2$, $\Im_3 = \{1,2,3,4,5,6,7,8\}$

$$\Im^3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\} \tag{33}$$

Assume the initial condition of system to be

$$\theta_0 = \theta_{-1} = \theta_{-2} = 1 \quad \tau_0 = \tau_{-1} = \tau_{-2} = 0,$$
$$d_0 = d_{-1} = d_{-2} = 1 \quad x(-2) = x(-1) = x(0) = [1 \quad -1.]^{\mathrm{T}}$$

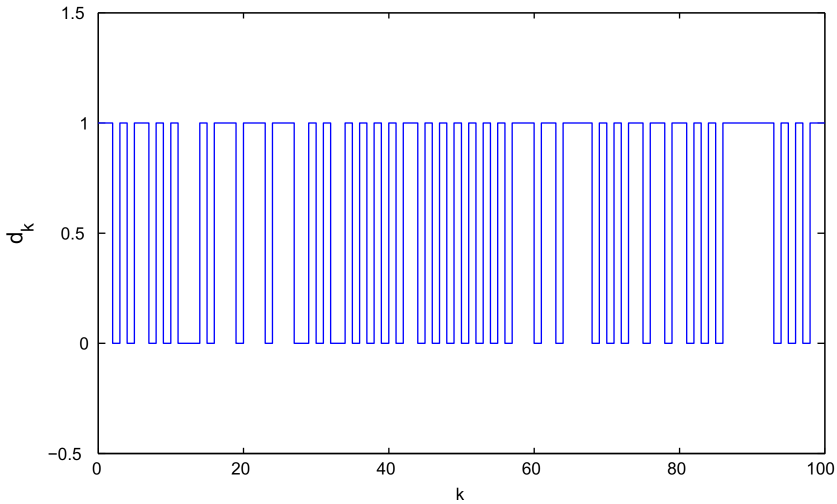

Fig. 4. Random mode d_k.

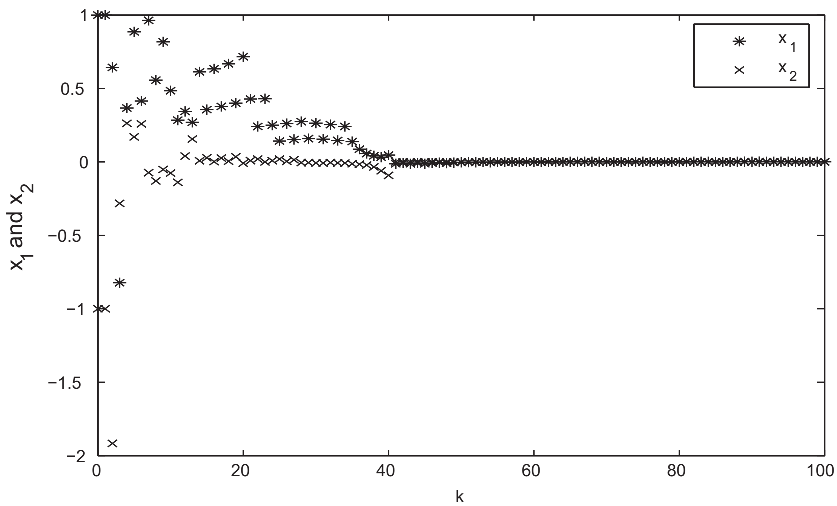

Fig. 5. State of the closed-loop system.

Applying the cone complementarity linearization algorithm, we can obtain controller gain vectors $K_1 = [\,-0.0273 \quad -0.0706\,]$, $K_2 = [\,0.0012 \quad -0.4486\,]$.

Figs. 2–4 illustrate the possible realizations of the Markovian jumping mode θ_k, d_k and τ_k, respectively. Under this mode sequence, the corresponding state trajectories of the closed-loop system is shown in Fig. 5. It is observed that the closed-lop system is stochastically stable.

5. Conclusions

This paper studies the stability problem of NCSs with Markovian characterization. By modelling the random delays as Markov chains, the closed-loop systems can be expressed as Markovian jump systems. The necessary and sufficient conditions of the stochastic stability are derived in the form of a set of linear matrix inequalities with matrix inversion constraints, from which the three-mode-dependent state-feedback gain can be solved by an iterative linear matrix inequality algorithm.

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